

Hash function based secret sharing scheme designs

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Abstract. Secret sharing schemes create an effective method to safeguard a secret by dividing it among several participants. By using hash functions and the herding hashes technique, we first set up a $(t + 1, n)$ threshold scheme which is perfect and ideal, and then extend it to schemes for any general access structure. The schemes can be further set up as proactive or verifiable if necessary. The setup and recovery of the secret is efficient due to the fast calculation of the hash function. The proposed scheme is flexible because of the use of existing hash functions.

Key words: Cryptographic hash function, herding hashes technique, secret sharing scheme, access structure.

1 Introduction

A secret sharing scheme has a strong motivation on private key protection. Based on Kerckhoffs's principle [1], only the private key in an encryption scheme is the secret and not the encryption method itself. When we examine the problem of maintaining sensitive information, we will consider two issues: availability and secrecy. If only one person keeps the entire secret, then there is a risk that the person might lose the secret or the person might not be available when the secret is needed. On the other hand the more people who can access the secret, the higher the chance the secret will be leaked. A secret sharing scheme (hereafter in this paper might be simply referred to as 'scheme') is designed to solve these issues by splitting a secret into shares and distributing these shares among a group of participants. The secret can only be recovered when the participants of an authorized subset join together to combine their shares.

Secret sharing schemes have applications in the areas of security protocols, for example, database security and multiparty computation (MPC). When a client wants to have his database outsourced (or so called "Database as a Service") to a third party, how to make sensitive information hidden from the server is a major concern. One common technique is to encrypt the data before storing it in the server. However, queries to the encrypted database are expensive. [2] suggested to use a threshold secret sharing scheme to split the data into different servers as shares to handle data privacy. MPC was first introduced in Yao's

seminal two millionaires's problem [3]. A secure MPC can be defined as n parties P_1, P_2, \dots, P_n join together to calculate a joint function $f(x_1, x_2, \dots, x_n)$, where x_i is the private input by P_i , $i = 1, \dots, n$. After the computation, each P_i will know the correct result of f but will not know other x 's. Secret sharing schemes play an important role in secure MPC as secrecy is highly required in such computations. For more MPC materials please refer to [4].

To summarize, a secret sharing scheme is a cryptographic primitive with many applications, such as PGP (Pretty Good Privacy) key recovering, visual cryptography, threshold cryptography, threshold signature, etc, in addition to those discussed above.

In this paper, we use the herding hashes technique to design a $(t + 1, n)$ threshold scheme which is perfect and ideal. Then, we show by examples of a hierarchical threshold scheme and a compartment scheme, that any general access structure can be realized. The resulting scheme can be further implemented as proactive easily. By adding an additional hash function we can make it verifiable. The setup is simple and the secret can be recovered quickly. The implementation is flexible as we can make use of existing hash functions.

The rest of paper is organized as follows. In Section 2 and Section 3 we review cryptographic hash functions and secret sharing schemes. Section 4 analyzes the complexity of the proposed scheme, and shows how to make the implementation practical. Then, we present several secret sharing scheme setups for illustration. In Section 5 we outline an implementation plan. In section 6 we conclude the paper and summarize the advantages of the proposed schemes.

2 Cryptographic hash functions

2.1 Iterative hash functions and multicollisions

A cryptographic hash function H takes an input message M of arbitrarily length and outputs a fixed-length string h . The output h is called the hash or message digest of the message M . It should be fast, preimage, second preimage and collision resistant. Please refer to the textbooks, such as [5,6], for the details.

An iterative hash function H is basically built from iterations of a compression function C using the Merkle-Damgård construction [7,8]. Briefly, the construction repeatedly applies the compression function as follows. (a) Pad the arbitrary length message M into multiple v -bit blocks: m_1, m_2, \dots, m_b . (b) Iterate the compression function $h_i = C(h_{i-1}, m_i)$, where h_i and h_{i-1} are intermediate hashes of u -bit strings, h_0 is the initial value (or initial vector) IV, and i ($1 \leq i \leq b$) is an integer. (c) Output h_b as the hash of the message M , i.e., $H(M) = h_b = C(h_{b-1}, m_b)$.

Suppose we apply the birthday attack to get b pairs of blocks $(m_1, m'_1), \dots, (m_b, m'_b)$ such that

$$h_i = C(h_{i-1}, m_i) = C(h_{i-1}, m'_i), i = 1, \dots, b. \quad (1)$$

By enumerating all possible combinations of these b -pairs blocks with each pair containing two choices, we can build up 2^b colliding messages as follows (see

Fig. 1). Since it takes $2^{u/2}$ steps for finding one pair of blocks, this process takes approximately $b \times 2^{u/2}$ steps. So, it is relatively easy to find multi-collisions in an iterative hash function. Please refer to [6,9] for the details.

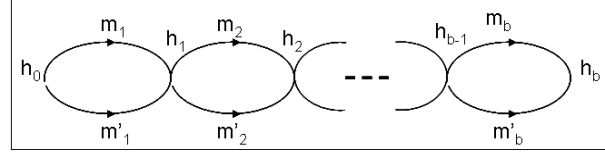


Fig. 1. Multicollisions in iterative hash functions.

2.2 Herding and Nostradamus attack

Kelsey and Kohno [10] have a detailed analysis of this attack. Stevens, Lenstra and Weger [11] applied the technique to predict the winner of the 2008 US Presidential Elections using a Sony PlayStation 3 in November 2007. We first build a large set of intermediate hashes at the first level: $h_{11}, h_{12}, \dots, h_{1w}$. Then message blocks are generated, so that they are linked and each intermediate hash at level 1 can reach the final hash, say h . This is called the diamond structure (see Fig. 2). We claim we can predict that something will happen in the future by announcing the final hash to the public. When the result is available, we construct a message as follows:

$$M = \text{Prefix} \| M^* \| \text{Suffix}, \quad (2)$$

where “Prefix” contains the results that we claimed we knew before it happens. M^* is a message block which links the “Prefix” to one of the intermediate hashes at level 1. “Suffix” is the rest of message blocks which linked the M^* to the final hash. In the example of Fig. 2, $M = \text{Prefix} \| M^* \| \text{Suffix}$, $\text{Suffix} = m_{15} \| m_{23} \| m_{32}$, and $H(M) = h_{41} = h$.

3 Secret sharing schemes

Based on a $(t+1, n)$ threshold scheme, many properties of secret sharing schemes can be easily demonstrated. It has a simple access structure and basis (Section 3.2). It is perfect and ideal (Section 3.3) and can be further implemented as proactive (Section 3.4) or verifiable (Section 3.5). Also, the distribution of the shares and recovery of the secret are through polynomial evaluation and polynomial interpolation, respectively, which are easy to follow.

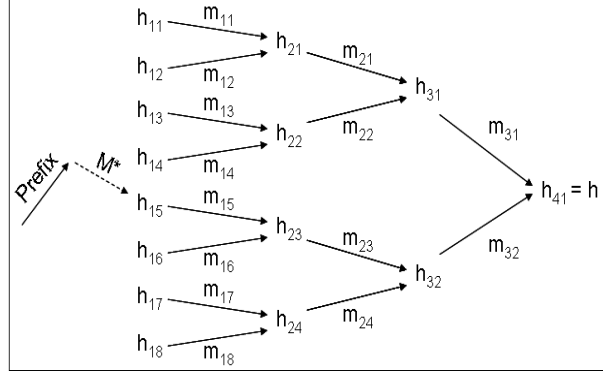


Fig. 2. A simplified diamond structure to illustrate Nostradamus attack.

3.1 A $(t + 1, n)$ threshold scheme

In 1979 Shamir [12] proposed a $(t + 1, n)$ threshold scheme, under which each of the n participants P_1, P_2, \dots, P_n receives a share of the secret and any group of $t + 1$ or more participants ($t \leq n - 1$) can recover the secret. Any group of fewer than $t + 1$ participants cannot recover the secret. The concept used by Shamir is based on Lagrange polynomial interpolation. We generate a polynomial of degree at most t over \mathbb{Z}_q , where q is a large prime number ($q > n \geq t + 1$). The coefficients, $a_t, \dots, a_1 \in \mathbb{Z}_q$, are generated randomly and $a_0 \in \mathbb{Z}_q$ is the secret.

$$P(x) = a_t x^t + a_{t-1} x^{t-1} + \dots + a_1 x^1 + a_0 \pmod{q}. \quad (3)$$

The dealer arbitrarily chooses different $x_i \in \mathbb{Z}_q - \{0\}$, $i = 1, 2, \dots, n$, and stores them in a public area. The corresponding shares $P(x_i) \pmod{q}$ are then calculated and distributed to the participants privately, so that each participant gets a share of the secret. By the polynomial interpolation given any $t + 1$ points the polynomial coefficients can be recovered, hence the constant term a_0 which is the secret. Note that we want the n points to be all different to each other and the coefficients must be from the field \mathbb{Z}_q to make sure we can recover the original polynomial. Here, we don't want to give out the point $P(0)$, because $P(0)$ is the secret itself.

3.2 Access structure

Continuing with the construction above, it is reasonable to assume that any number of greater than $t + 1$ participants can always recover the secret. We call this property monotone. A group of participants, which can recover the secret when they join together, is called an authorized subset. In a $(t + 1, n)$ threshold scheme, any group of $t + 1$ or more participants forms an authorized subset. On the other hand, any group of participants that cannot recover the secret is called an unauthorized subset. An access structure Γ is a set of all authorized subsets.

Given any access structure Γ , $A \in \Gamma$ is called a minimal authorized subset if $B \subsetneq A$ then $B \notin \Gamma$. We use Γ_0 , for the basis of Γ , to denote the set of all minimal authorized subsets of Γ . In a $(t+1, n)$ threshold scheme, let P be the set of the participants:

$$\Gamma = \{A \mid A \subseteq P \text{ and } |A| \geq (t+1)\}, \quad (4)$$

$$\Gamma_0 = \{A \mid A \subseteq P \text{ and } |A| = (t+1)\}. \quad (5)$$

In secret sharing, we first define the access structure. Then, we realize the access structure by a secret sharing scheme. For instance, Shamir's $(t+1, n)$ threshold scheme realizes the access structure defined in Eq.4.

3.3 Perfect and ideal scheme

Shamir's scheme does not allow partial information to be given out even up to t participants joining together [5]. A scheme with such a property is called a perfect secret sharing scheme. Based on information theory, the length of any share must be at least as long as the secret itself in order to have perfect secrecy. The argument for this is that up to t participants have zero information under the perfect sharing scheme, but when one extra participant joins the group, the secret can be recovered. That means any participant has his share at least as long as the secret. If the shares and the secret come from the same domain, we call it an ideal secret sharing scheme. In this case, the shares and the secret have the same size.

3.4 Proactive scheme

In a secret sharing scheme, we need to consider the possibility that an active adversary may find out all the shares in an authorized subset to discover the secret eventually if he is allowed to have a very long time to gather the necessary information. In order to prevent this from happening, we refresh and redistribute new shares to all the participants periodically. After finishing this phase, the old shares are erased safely. The secret remains unchanged. By doing so, the information gathered by the adversary between two resets would be useless. In order to break the system an adversary has to get enough information regarding the shares within any two periodic resets.

Based on Shamir's scheme, Herzberg, Jarecki, Krawczyk, and Yung [13] derived a proactive scheme, which uses the following method to renew the shares. In addition to $P(x)$ of Eq.3, the dealer generates another polynomial $Q(x)$ of degree at most t over \mathbb{Z}_q without the constant term (i.e., $b_0 = 0$),

$$Q(x) = b_t x^t + b_{t-1} x^{t-1} + \dots + b_1 x \pmod{q}, \quad (6)$$

where $b_1, \dots, b_t \in \mathbb{Z}_q$. Then add $P(x)$ and $Q(x)$ together to get the sum $R(x)$ as

$$R(x) = c_t x^t + c_{t-1} x^{t-1} + \dots + c_1 x + a_0 \pmod{q}, \quad (7)$$

where $c_i = a_i + b_i \pmod{q}$ for $i = 1, \dots, t$.

The dealer then sends out new shares $R(1), R(2), \dots, R(n)$ to the n participants to replace the old shares $P(0), P(1), \dots, P(n)$. It remains a $(t + 1, n)$ threshold scheme with the same original secret.

The above technique can be extended so that all the participants can engage in the shares renewal process. This method can eliminate the situation where all the work is done by the dealer, and the scheme will be more secure.

3.5 Verifiable scheme

In reality, we need to consider the situation that the dealer or some of the participants might be malicious. In this case, we need to set up a verifiable secret sharing scheme so that the validity of the shares can be verified. Here we discuss Feldman's scheme [14] which is a simple verifiable secret sharing scheme based on Shamir's scheme. Also see [15] for another reference.

The idea is to find a cyclic group G of order q where q is a prime. Since it is cyclic, a generator of G , say g , exists. As other cryptographic protocols, we assume the parameters of G are carefully chosen so that the discrete logarithm problem is hard to solve in G . Let p, q be primes such that q divides $(p-1)$, $g \in \mathbb{Z}_p^*$ of order q . The dealer generates a polynomial $P(x)$ over \mathbb{Z}_q of degree at most t as shown in Eq.3, and sends out $P(i)$ to participant i as before. In addition to this, he also broadcast in a public channel the commitments: $g^{a_i} \pmod{p}$, $i = 1, \dots, n$. Each participant i will verify if the following equation is true.

$$g^{P(i)} = (g^{a_0})(g^{a_1})^i(g^{a_2})^{i^2} \dots (g^{a_t})^{i^t} \pmod{p}, i = 1, \dots, n. \quad (8)$$

Based on the homomorphic properties of the exponentiation, the above condition will hold true if the dealer sends out consistent information. Later, when the participants return their shares for secret recovering, the dealer can also verify their shares by the same method. Feldman's scheme is not perfect since partial information about the secret, g^{a_0} , is leaked out. However we assume it is difficult to get the secret a_0 from g^{a_0} if the discrete logarithm problem is hard to solve under G .

4 Hash function based secret sharing scheme designs

4.1 Related work

Zheng, Hardjono and Seberry [16] discuss how to reuse shares in a secret sharing scheme by using the universal hash function. Chum and Zhang [17,18] show how to apply hash functions to Latin square based secret sharing schemes for improvements. In this paper, we extend idea of herding and Nostradamus attacks [10] to any secret sharing scheme. We propose how to speed up the process and hence make it practical. An outline of the implementation is also suggested.

One direction for research in secret sharing schemes is to reduce the size of the shares. One approach is to use a ramp scheme [19,20]. However, the limitation

of a ramp scheme is that it leaks partial information. If we want the scheme to be perfect as aforementioned, the size of the share should be at least as long as the size of the secret. It has been shown that there are no ideal schemes for certain access structures. Please refer to [21,22,23] for examples. That means at least one participant needs to hold a share whose size is longer than the secret. Here we want to set up an ideal scheme for any access structure with the aid of a public area which is justified, because of its relatively low cost to maintain. As long as no one can change or destroy the public area it will work. To be explained later, the public area does not help any authorized subset, with just one participant missing, to recover the secret is easier than an outsider if the length of the share is the same as that of the hash. From the public area, we can only identify which group of participants can be joined together to recover the secret. In general, this is not a problem. In reality, should a secret need to be accessed, we know who should be contacted. Our scheme is flexible and fast because it makes use of the properties of the existing hash functions.

4.2 A simplified diamond structure

In the proposed new scheme we set up one message M_{priv} for one authorized subset. After building a diamond structure, all the M_{priv} 's will be herded to a final hash h , which is the secret. That means any authorized subset can recover the secret by their private shares and the corresponding public information (see Fig. 3). More details are in the next section.

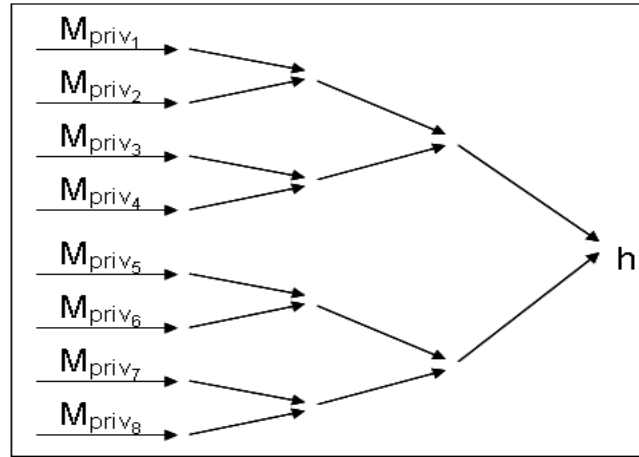


Fig. 3. Any authorized subset will herd to the final hash, i.e. the secret.

Based on the birthday attack the complexity of building a diamond structure for our scheme is exponential, too expensive to implement. We will show how

to avoid such complexity and make the scheme efficient and practical in next section.

4.3 Newly proposed scheme

A. Setup

(a) We randomly generate a share of the same size as that of the hash to each participant. Suppose there are n participants, then share s_i will be assigned to participant P_i , $i = 1, \dots, n$.

(b) We determine all the minimal authorized subsets. Suppose we have A_1, \dots, A_w minimal authorized subsets. Each participant holds a share and combination of the shares of any one of these w authorized subsets will form a private message M_{priv} . The combination will be the concatenation of the shares in participant sequence. For example, if an authorized subset consists of P_1, P_3 and P_4 , then $M_{priv} = s_1 || s_3 || s_4$.

(c) Calculate the hashes for the following

$$H(M_{priv_i}) = h_i, \quad i = 1, \dots, w. \quad (9)$$

Let h be the secret and of the same size of h_i . If we want the secret to be random, we can set h to one of the h_i . Or h is a pre-determined fixed secret. We continue to generate a control c_i as follows (here \oplus is bitwise exclusive OR):

$$c_i = h_i \oplus h, \quad i = 1, \dots, w. \quad (10)$$

To summarize, after the setup process each participant P_i gets a random share s_i , $i = 1, \dots, n$. Public information c_i , where $i = 1, \dots, w$, is generated. Control area c_i 's help to herd all the intermediate hashes h_i 's to the final hash h . This eliminates the complexity of building a diamond structure.

B. Secret recovering

Suppose authorized subset A_i consists of participants P_1, \dots, P_b . Joining together they can recovery the secret as follows, see Fig. 4.

- 1) Get the public information c_i .
- 2) $H(s_1 || s_2 || \dots || s_b) = h_i$, and $h_i \oplus c_i = h$.

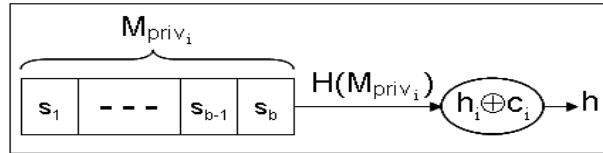


Fig. 4. Secret recovery by combination of private and public information.

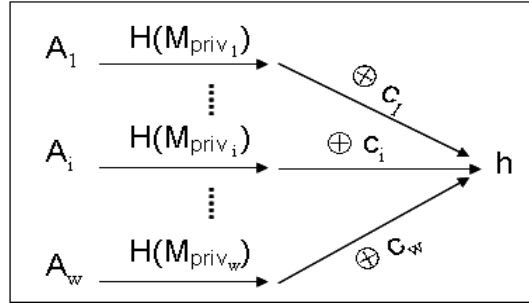


Fig. 5. Secret recovery for any authorized subset.

This applies to any authorized subset, see Fig. 5.

C. Performance

In the setup step the operations involved are generation of random shares s_1, \dots, s_n , calculation of hashes $h_i = H(M_{priv_i})$, and generation of control area $c_i, i = 1, 2, \dots, w$. In the secret recovering step, assuming participants of authorized subset A_i join together, we just need to calculate the hash (i.e., secret) by $h = H(M_{priv_i}) \oplus c_i$. All the operations during the setup and secret recovering are efficient. This makes the proposed scheme practical.

D. Properties of the proposed scheme

- a) Perfect: Based on randomness of a hash function, any participant cannot figure out any information about the hash from his/her share. Suppose a participant in a minimal authorized subset is missing, the randomness property makes it impossible to recover his/her share directly. Brute force is the only way to determine the share of the missing participant. However, the rest of the participants cannot rule out any possibility of the value of the share, as each guessed value can be combined with their shares come up to a valid hash. So, in the worst case, they need to try $2^{|s|}$ times. On the other hand, an outsider needs to try $2^{|h|}$ times. If we choose the size of the share s as same as that of the hash h , any authorized subset with just one participant missing does not have any additional information to help them do better than any outsider.
- b) Ideal: Each participant holds one share which has the same size of the hash. The smaller the size of the shares $|s|$, the more efficient the scheme would be. However, as discussed above, any authorized subset with just one participant missing can recover the hash by trying at most $2^{|s|}$ times. That means they can break the system more easily than an outsider if $|s|$ is smaller than $|h|$. On the other hand, it will not increase the security level by setting $|s|$ larger than $|h|$. By brute force, any outsider can try at most $2^{|h|}$ times to recover the secret.

- c) Fast setup and recovery of the secret: The calculation of hash function is fast. No complicated or intensive computation, such as polynomial evaluation/interpolation, is needed.
- d) Application of minimal authorized subset: As we explained earlier, we can speed up the whole process by considering the minimal authorized subset only.
- e) General access structure: As we shall see in the following examples, this approach can be extended to any general access structure.
- f) Flexible: A hash function can handle any message of arbitrary length so there is no limit to the number of participants. We can always change to a new and better hash function should it become available. For example, we use SHA-2 now, when SHA-3 is available we can switch to it.
- g) No special hardware or software is required: For example, no need to handle a large number or find a large prime, etc.

4.4 Set up an ideal perfect $(t + 1, n)$ threshold scheme

As we mentioned before, a $(t + 1, n)$ threshold scheme has a simple access structure. Based on the monotone property, we only need to consider $N = C(n, t + 1)$ minimal authorized subset only. Here,

$$N = C(n, t + 1) = \frac{n!}{(t + 1)!(n - t - 1)!}. \quad (11)$$

Example: A $(2, 3)$ threshold scheme

Let s_1, s_2 , and s_3 be shares of participants P_1, P_2 , and P_3 , respectively. Then, the access structure consists of three ($N = 3$ by the Eq.11) minimal authorized subsets A_1, A_2 and A_3 . The controls c_1, c_2, c_3 will be stored in the public area, see Fig. 6.

- a) $A_1 : \{P_1, P_2\}$ $s_1 || s_2; c_1$
- b) $A_2 : \{P_1, P_3\}$ $s_1 || s_3; c_2$
- c) $A_3 : \{P_2, P_3\}$ $s_2 || s_3; c_3$

4.5 Set up an ideal perfect scheme for general access structure

Our herding hashes technique discussed above can be used to set up a secret sharing scheme for any general access structure. Here, we illustrate a hierarchical threshold scheme and a compartment scheme as follows.

Hierarchical threshold scheme The following is the conjunctive hierarchical scheme proposed by Tassa [24]. Let U be the set of n participants. U is divided into m levels:

$$U = U_1 \cup U_2 \cup \dots \cup U_m \text{ and } U_i \cap U_j = \emptyset, \forall i, j : 1 \leq i < j \leq m. \quad (12)$$

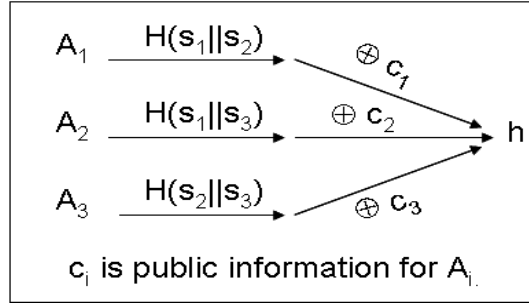


Fig. 6. A (2, 3) threshold scheme example.

Instead of just assigning a threshold number k as a regular secret sharing scheme, a set of numbers $k = \{k_1, \dots, k_m\}$ in a strictly increasing order is set up: $0 < k_1 < k_2 < \dots < k_m$. Then, the (k, n) hierarchical threshold access structure is:

$$T = \{V \subset U \mid |V \cap (U_1 \cup \dots \cup U_i)| \geq k_i, \forall i \in \{1, \dots, m\}\}. \quad (13)$$

So if V is an authorized subset, then:

- the number of participants in V at level 1 $\geq k_1$
- AND the number of participants in V at level 1, 2 $\geq k_2$
- AND
- AND the number of participants in V at level 1, ..., $m \geq k_m$.

If we just require any one of the above conditions to be true at any level, we can simply change AND to OR, then, we will get a disjunctive hierarchical secret sharing scheme which is originally proposed by Simmons [25].

Example: Conjunctive hierarchical secret sharing scheme

Let $U = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ be the set of the participants. There are three levels, $U_1 = \{P_1, P_2\}$ for level 1, $U_2 = \{P_3, P_4\}$ for level 2, $U_3 = \{P_5, P_6\}$ for level 3, and $\{k_1, k_2, k_3\} = \{1, 2, 3\}$. Based on Γ_0 , the set of minimal authorized subsets, we have the following setup, where s_i is the corresponding share for P_i and c_i 's are the corresponding public information, A_i 's are authorized subsets.

- | | |
|------------------------------|--------------------------|
| a) $A_1 : \{P_1, P_3, P_5\}$ | $s_1 s_3 s_5; c_1$ |
| b) $A_2 : \{P_1, P_3, P_6\}$ | $s_1 s_3 s_6; c_2$ |
| c) $A_3 : \{P_1, P_4, P_5\}$ | $s_1 s_4 s_5; c_3$ |
| d) $A_4 : \{P_1, P_4, P_6\}$ | $s_1 s_4 s_6; c_4$ |
| e) $A_5 : \{P_1, P_3, P_4\}$ | $s_1 s_3 s_4; c_5$ |
| f) $A_6 : \{P_2, P_3, P_5\}$ | $s_2 s_3 s_5; c_6$ |
| g) $A_7 : \{P_2, P_3, P_6\}$ | $s_2 s_3 s_6; c_7$ |

h) $A_8 : \{P_2, P_4, P_5\}$	$s_2 s_4 s_5; c_8$
i) $A_9 : \{P_2, P_4, P_6\}$	$s_2 s_4 s_6; c_9$
j) $A_{10} : \{P_2, P_3, P_4\}$	$s_2 s_3 s_4; c_{10}$
k) $A_{11} : \{P_1, P_2, P_3\}$	$s_1 s_2 s_3; c_{11}$
l) $A_{12} : \{P_1, P_2, P_4\}$	$s_1 s_2 s_4; c_{12}$
m) $A_{13} : \{P_1, P_2, P_5\}$	$s_1 s_2 s_5; c_{13}$
n) $A_{14} : \{P_1, P_2, P_6\}$	$s_1 s_2 s_6; c_{14}$

Compartment scheme Compartment scheme [25] works as follows. Let U be the set of n participants, and U is divided into m compartments: $U = U_1 \cup U_2 \cup \dots \cup U_m$ and $U_i \cap U_j = \emptyset$ for all $i, j : 1 \leq i < j \leq m$.

There is a threshold assigned to each group, say t_1 for U_1 , t_2 for U_2 , etc. An authorized subset will:

- a) contain at least t_i participants in U_i (an individual threshold scheme for group U_i);
- b) contain at least t participants (an overall threshold scheme).

Example: Compartment secret sharing scheme

Let $U = \{P_1, P_2, \dots, P_6\}$ be the set of the participants, three compartments $U_1 = \{P_1, P_2\}$, $U_2 = \{P_3, P_4\}$ and $U_3 = \{P_5, P_6\}$. We want at least 1 participant from each compartment and 4 participants overall. Once we determine the T_0 , the implementation will be straightforward.

a) $A_1 : \{P_1, P_2, P_3, P_5\}$	$s_1 s_2 s_3 s_5; c_1$
b) $A_2 : \{P_1, P_2, P_3, P_6\}$	$s_1 s_2 s_3 s_6; c_2$
c) $A_3 : \{P_1, P_2, P_4, P_5\}$	$s_1 s_2 s_4 s_5; c_3$
d) $A_4 : \{P_1, P_2, P_4, P_6\}$	$s_1 s_2 s_4 s_6; c_4$
e) $A_5 : \{P_1, P_3, P_4, P_5\}$	$s_1 s_3 s_4 s_5; c_5$
f) $A_6 : \{P_1, P_3, P_4, P_6\}$	$s_1 s_3 s_4 s_6; c_6$
g) $A_7 : \{P_2, P_3, P_4, P_5\}$	$s_2 s_3 s_4 s_5; c_7$
h) $A_8 : \{P_2, P_3, P_4, P_6\}$	$s_2 s_3 s_4 s_6; c_8$
i) $A_9 : \{P_1, P_3, P_5, P_6\}$	$s_1 s_3 s_5 s_6; c_9$
j) $A_{10} : \{P_1, P_4, P_5, P_6\}$	$s_1 s_4 s_5 s_6; c_{10}$
k) $A_{11} : \{P_2, P_3, P_5, P_6\}$	$s_2 s_3 s_5 s_6; c_{11}$
l) $A_{12} : \{P_2, P_4, P_5, P_6\}$	$s_2 s_4 s_5 s_6; c_{12}$

4.6 Set up a verifiable scheme for general access structure

Let f, g be cryptographic hash functions. The dealer generates shares s_1, s_2, \dots , and distributes each share to each participant and then publishes the hashes (by hash function g) of each share as commitments: g_1, g_2, \dots . Participant i verifies his or her share by checking if $g(m_i) = g_i$ holds. If all participants confirm that taking his or her share as input to the hash function g , he or she gets the hash value equal to one of the commitments published by the dealer, we conclude the

dealer sends out consistent shares. Likewise, when the participants return their shares, the dealer can verify in the same way.

Hash function g is used to make the scheme verifiable. Hash function f is used as H in 4.3 for the scheme. Partial information was given out here, however, if g is preimage resistant, it would be infeasible to find the original share s_i from g_i . Participant i can fool the party if he or she can find s'_i such that $g(s_i) = g(s'_i) = g_i$. However, this is also extremely difficult to achieve if g is second preimage resistant.

4.7 Set up a proactive scheme

We pick up any authorized subset to recover the secret h , then repeat the process to generate and re-distribute new shares s'_1, s'_2, \dots . Based on the secret h and the newly generated shares s'_1, s'_2, \dots , we determine and update the new public control information c'_1, c'_2, \dots . Finally we delete the secret h . So shares are refreshed and the secret remains unchanged.

5 Implementation plan

Suppose there are n participants P_1, \dots, P_n and w minimal authorized subsets A_1, \dots, A_w for a given access structure. Let H be the hash function for the implementation. The secret stores in a variable h , which has the same size as the output hash of H .

- (a) If the secret is fixed, input and store it in h . Otherwise skip this step.
- (b) FOR $i = 1, \dots, n$
 - Generate randomly s_i for P_i
- ENDFOR
- (c) FOR $i = 1, \dots, w$
 - Construct M_{priv_i} based on shares of participants in A_i in participant sequence
 - $h_i = H(M_{priv_i})$
 - If $i = 1$ and h is empty, then $h = h_1$ /* If no input secret, set the secret to the first randomly generated intermediate hash h_1 . */
 - $c_i = h_i \oplus h$
 - K_i = concatenation of the ordered indices of participants in A_i
 - Write c_i in public area based on key K_i
- ENDFOR
- (d) FOR $i = 1, \dots, n$
 - Send s_i to P_i privately
- ENDFOR

(e) Delete all s_i (shares) and the h (secret).

After the implementation:

- 1) We create the following (see Fig. 7)
 - i) private shares for participants: s_1, \dots, s_n .
 - ii) public information c_1, \dots, c_w .
- 2) Any authorized subset A_i can form key K_i to get the corresponding c_i to recover the secret (see 4.3B).

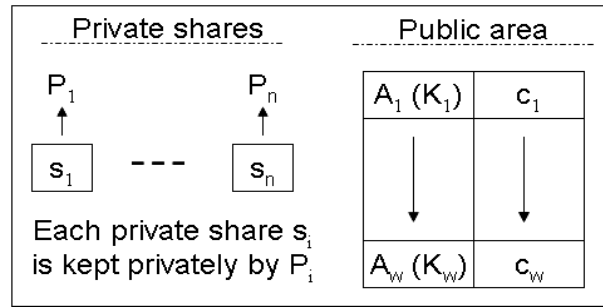


Fig. 7. Shares for participants and public area.

6 Conclusion

This paper shows how to design various secret sharing schemes based on cryptographic hash functions so that any general access structure can be realized as perfect and ideal. The implementation is simple and efficient as we make use of the existing hash functions. The share distribution and secret recovery can be done quickly due to fast calculation of hash functions. We can further implement these schemes as proactive and, or verifiable if required.

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References

1. J. Katz and Y. Lindell. *Introduction to Modern Cryptography*. Chapman and Hall/CRC, 2007.

2. D. Agrawal, A. Abbadi, F. Emekci, and A. Metwally. Database management as a service: challenges and opportunities. In *IEEE ICDE*, 2009.
3. A.C. Yao. Protocols for secure computations (extended abstract). In *the 21st Annual IEEE Symposium on the Foundations of Computer Science*, pages 160–164, 1982.
4. R. Cramer, I. Damgård, and J.B. Nielsen. Multiparty Computation, an Introduction. Lecture Notes. Available <http://www.brics.dk/~jbn/smc.pdf>, 2009.
5. D. Stinson. *Cryptography, Theory and Practice*. Chapman and Hall/CRC, 3rd edition, 2005.
6. W. Trappe and L. Washington. *Introduction to Cryptography with Coding Theory*. Prentice Hall, 2nd edition, 2006.
7. I. Damgård. A design principle for hash functions. In *Proc. of CRYPTO 1989*, volume 435 of *LNCS*.
8. R. C. Merkle. *Secrecy, Authentication, and Public Key Systems*. PhD thesis, Stanford University, 1979.
9. A. Joux. Multicollisions in iterated hash functions. Application to cascaded construction. In *Proc. of CRYPTO 2004*, volume 3152 of *LNCS*, pages 306–316.
10. J. Kelsey and T. Kohno. Herding hash functions and the Nostradamus attack. Cryptology ePrint Archive, Report 2005/281, 2005.
11. M. Stevens, A.K. Lenstra, and B. Weger. Predicting the winner of the 2008 US presidential elections using a Sony PlayStation 3. Available online <http://www.win.tue.nl/hashclash/Nostradamus>, November 2007.
12. A. Shamir. How to share a secret. *Communications of the ACM*, 22(11):612–613, 1979.
13. A. Herzberg, S. Jarecki, H. Krawczyk, and M. Yung. Proactive secret sharing. In *Proc. of CRYPTO 1995*, volume 963 of *LNCS*, 1995.
14. P. Feldman. A practical scheme for non-interactive verifiable secret sharing. In *Proc. of the 28th IEEE Symposium on the Foundations of Computer Science*, pages 427–437, 1987.
15. L. Harn and C. Lin. Detection and identification of cheaters in (t, n) secret sharing scheme. *Designs, Codes and Cryptography*, 52(1):15–24, 2009.
16. Y. Zheng, T. Hardjono, and J. Seberry. Reusing shares in secret sharing schemes. *Computer Journal*, 37(3):199–205, 1994.
17. C. Chum and X. Zhang. Applying hash functions in the Latin square based secret sharing schemes. In *Proc. of The 2010 International Conference on Security and Management (SAM'10)*, pages 197–203, 2010.
18. C. Chum and X. Zhang. The Latin squares and the secret sharing schemes. *J. of Groups - Complexity - Cryptology*, 2:175–202, 2010.
19. M. Iwamoto and H. Yamamoto. Strongly secure ramp secret sharing schemes for general access structures. *Journal Information Processing Letters*, 97(2), 2006.
20. J. Kurihara, S. Kiyomoto, K. Fukushima, and T. Tanaka. A fast (k, l, n) -threshold ramp secret sharing scheme. *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, E92.A(8):1808–1821, 2009.
21. J. Benaloh and J. Leichter. Generalized secret sharing and monotone functions. In *CRYPTO 1988*, volume 403 of *LNCS*, pages 27–35, 1990.
22. R.M. Capocelli, A. De Santis, L. Gargano, and U. Vaccaro. On the size of shares for secret sharing schemes. *Journal of Cryptology*, 6(3):157–167, 1993.
23. L. Csirmaz. The size of a share must be large. *Journal of Cryptology*, 10(4):223–231, 1997.
24. T. Tassa. Hierarchical threshold secret sharing. *Journal of Cryptology*, 20(11):237–264, 2007.

25. G.J. Simmons. How to (really) share a secret. In *CRYPTO1988*, volume 403 of *LNCS*, pages 390–448, 1990.